Printed Pages – 7

नोट : प्रथम प्रश्नपत्र हल करना अनिवार्य है। वैकल्पिक ग्रुप में जहाँ 01 से अधिक प्रश्नपत्र है उनमें से 01 प्रश्नपत्र हल करना अनिवार्य है। कुल 06 ग्रुप में से कोई 04 ग्रुप के प्रश्न पत्र हल करना अनिवार्य है। कुल 05 प्रश्नपत्र हल करना अनिवार्य है।

Note: *Each section is compulsorily written on separate answer sheet.*

H-2561

M. A. / M. Sc. (Fourth Semester) Examination, 2021

MATHEMATICS

Paper : First (Compulsory)

(Functional Analysis-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *Attempt all questions. Each question carry equal marks. Each question must be answered in maximum 800 words.*

1. Let X be a normed space and let $x_0 \neq 0$ be any element of x, then prove that there exists a bonded linear functional $\tilde{f}$ on x such that $\| \tilde{f} \| = 1$, $\tilde{f}(x_0) = \| x_0 \|$.

2. Let X be a normed space and let

(i) The sequence $(\| X_n \|)$ is bounded.

(ii) For every element f of a total subset $M \subset X^1$ we have $f(x_n) \to f(x)$, then prove that $x_n \xrightarrow{w} x$

3. Let X be an inner product space and $M \neq \phi$ is a convex subset which is complet subspace y and $x \in X$ fixed. Then prove that $z = x - y$ is orthogonal to y.

4. Let (e_k) be an orthonormal sequence in an inner product space X. Then for every $x \in X$, prove that $\sum_{k=1}^{\infty} | < x, e_k > |^2 \leqq \| x \|^2$.

5. If H_1 and H_2 Hilbert spaces and $h : H_1 \times H_2 \to K$ a bounded sesquilinear form. Let $S : H_1 \to H_2$ be a bounded linear operator, then prove that h has a representation $h(x, y) = < S_x\ y >$.

and S is uniquely determined by h and has the norm $\| s \| = \| h \|$

H-2562

M. A. / M. Sc. (Fourth Semester) Examination, 2021

Section- A : Paper : First (Optional Group-I)

ADVANCED FUNCTIONAL ANALYSIS-II

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *Attempt all questions. Each questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Let (V, τ) be a topological linear space over ϕ. Then prove that following are equivalent :

(i) (V, τ) is finite dimensional.

(ii) There exists a compact set $B \subset V$ with a non empty interior such that the origin belongs to the interior of B.

2. State and prove closed graph theorem for Frehet spaces.

3. Let V is a topological linear space on which V^* separates points. It K is a convex set in V, then prove that K is the closed convex hull of the set of its extreme points.

4. Let $(V, \|\cdot\|)$ be a normed linear space over ϕ and suppose $T : V \to V^{**}$ is defined by $T(x)(x^*) = x^*(x)$, $x \in V$, $x^* \in V^*$. Then prove that T is isometric isomorphism.

5. Let B be a bounded set of $D(\Omega)$. Then prove that there exists a compact subset K such that

$$\text{supp}(\phi) \subseteq K \text{ for every } \phi \in B \text{ and } \sup_{x \in X,\, \phi \in B} \left| D^j \phi(x) \right| < \infty$$

for every differentiate operator D^j.

Or

H-2563

M. A. / M. Sc. (Fourth Semester) Examination, 2021

Section- B Paper : Second (Optional Group-I)

MECHANICS-II

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. Each questions carry equal marks. Each question must be answered in maximum 800 words.*

1. State and prove theorem on total energy.

2. Derive Euler's euqation for one dependent function.

3. State and prove principle of least action.

4. Define Lagrange bracket and show that the Lagrange bracket is invariant under canonical transformation.

5. A particle moves in the XY-plane under the influence of a central force depending only on its distance from the origin. Explain.

H-2564

M. A. / M. Sc. (Fourth Semester) Examination, 2021

Section-A Paper : First (Optional Group-II)

ABSTRACT HARMONIC ANALYSIS-II-2

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Let $f_1 , f_2 \in C_0^+ (G), g \neq 0$, then show that

$$I_g (f_1 + f_2) \leq I_g (f_1) + I_g (f_2)$$

2. Prove that translation is uniformly continuous in L_p-norm.
3. Prove that collection of all characters of G forms a LCA group G is also dual of G.
4. Prove that fourier transform is norm decreasing and therefore continuous.
5. State and prove Duality theorem.

Or

H-2565

M. A. / M. Sc. (Fourth Semester) Examination, 2021

Section-B Paper : Second (Optional Group-II)

ADVANCE SPECIAL FUNCTION-II-4

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Prove that $\frac{d}{dx} \left(x J_n J_{n+1} \right) = x \left(J_n^2 - J_{n+1}^2 \right)$.
2. Prove that $P_n (x) = \frac{1}{2^n n!} D^n \left\{ \left(x^2 - 1 \right)^n \right\}$
3. State and prove Murphy's formula.
4. Prove that $e^{2xt - t^2} = \sum_{n=0}^{\infty} \frac{H_n (x) t^y}{n!}$.
5. Prove that $(n+1) L_{n+1} (x) = (2n + 1 - x) L_n (x) - x L_{n-1} (x)$.

Or

H-2566

M. A. / M. Sc. (Fourth Semester) Examination, 2021

Section-C Paper : Third (Optional) Group-II

ADVANCE GRAPH THEORY-II-3

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Define planner and non-planner graph. Prove that a complete graph of five vertices is non-planner.
2. "A graph has a dual if and only if it is planar." Explain.
3. State and prove Polya's counting theorem.
4. Define cut-set. Prove that in a connected graph G every cut-set must contain atleast one branch of every spanning tree of grpah G.
5. Explain shortest spanning tree and also give it's applications. State and prove Kruskal's algorithm.

H-2567

M. A. / M. Sc. (Fourth Semester) Examination, 2021

(Optional Group - III)

MATHEMATICS

Paper : First

(Theory of Linear Operators-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Let $T : H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space H. Then :

   (a) T has the spectral representation

   $$T = \int_{m=0}^{M} \lambda \, dE\lambda,$$

   where (E_λ) is the spectral family associated with T, the integral is to be uniform operator convergence.

   [Convergence is the norm on B (H, H)] and for all $x, y \in H$, $\langle T_{x,y} \rangle = \int_{m=0}^{M} \lambda \, dw(\lambda)$, $w(\lambda) = \langle E_\lambda x, y \rangle$.

   where the integral is on ordinary Riemann Stieltje's integral.

   (b) If p is a polynomial in λ with real coefficient, say $p(\lambda) = \alpha_n \lambda^n + \alpha_{n-1}\lambda^{n-1} + ... + \alpha_0$, then the operator p (T) defined by

   $$p(T) = \alpha_n T^n + \alpha_{n-1} T^{n-1} + ... + \alpha_0 I$$

has the spectral representation

$$p(T)=\int_{m=0}^{M} p(\lambda)\,d E_{\lambda}$$

and for all $x,\ y \in H$, $\langle p(T)x,\ y\rangle = \int_{m=0}^{M} p(\lambda)\,dw(\lambda),\ w(\lambda)=\langle E_{\lambda}x,\ y\rangle$.

2. If E is a compact and regular spectral measure with spectral $\wedge$ and if f is a complex value continuous function on X. Then $\| f\, dE \| = N_E(f)$.

3. State and prove that Hellinger-Toeplitz theorem.

4. For a symmetric linear operator $T : \mathcal{D}(T) \to H$ where H is a complex Hilbert space and $\mathcal{D}(T)$ is dense in H. We have $(\tilde{T})^{*} = T^{*}$.

5. State and prove that spectral theorem for unitary operator.

H-2568

M. A. / M. Sc. (Fourth Semester) Examination, 2021

(Optional Group-IV)

MATHEMATICS

Paper : First (Optional Group-IV)

(Operations Research-II)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. For the given information :

Activity	0 – 1	1 – 2	1 – 3	2 – 4	2 – 5	3 – 4	3 – 6	4 – 7	5 – 7	6 – 7
Duration	2	8	10	6	3	3	7	5	2	8

(i) Identify the critical path and find the total project duration.

(ii) Determinetotal, free and independent float.

2. Use dynamic programming to solve

Maximize $\quad Z = y_1 \cdot y_2 \cdot y_3$

Subject to

$$y_1 + y_2 + y_3 = 5$$

$$y_1, y_2, y_3 \geq 0$$

3. Solve the game :

Player B

$$\text{Player A}\begin{bmatrix} 1 & 2 \\ 5 & 4 \\ -7 & 9 \\ -4 & 3 \\ 2 & 1 \end{bmatrix}$$

4. Using Lagrangian multipliers solve the NLPP

Minimize $Z = 6x_1^2 + 5x_2^2$

Subject to

$x_1 + 5x_2 = 3$

$x_1, x_2 \geq 0$

5. Explain in brief how quadratic programming differ from linear programming problem.

H-2569

M. A. / M. Sc. (Fourth Semester) Examination, 2021

MATHEMATICS

Paper : First

(Optional Group-V)

(Integral Transforms-II-4)

Maximum Marks : 40 (Regular) / 50 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Solve : $\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial t^2}$, if $y(0, t) = 0 = y(5, t)$, $y(x, 0) = 10\sin 4\pi x$.

2. A cantilever beam, clamped at $x = 0$ and free at $x = l$, carries a uniform load W_0 per unit length. Show that the deflexion is $y(x) = \frac{W_0 x^3}{24\,El}\left(x^2 - 4lx + 6l^2\right)$. Where E is Young's modulus of elasticity.

3. Find Fourier sine transform of $f(x) = \frac{1}{x}$.

4. If $f(s)$ is the Fourier transform of $F(x)$ then find the Fourier transform of $F(ax)$.

5. Find the finite cosine transform of $f(x) = \sin(nx)$.

F-2570

M. A. / M. Sc. (Fourth Semester) Examination, 2021

(Optional Group - VI)

MATHEMATICS

Paper : First

(Fundamentals of Computer Science - II)

Maximum Marks : 25 (Regular) / 35 (Private)

Note: *All questions are compulsory. All questions carry equal marks. Each question must be answered in maximum 800 words.*

1. Explain Hybride Inheritance and single Inheritance with examples.

2. Differentiate between put ( ) and get ( ) function.

3. What factors are used to classify the database systems?

4. Explain views. How views are created? Explain it with application.

5. Differentiate between Network and Distributed System.